Experimental Use Licensing with Non-drastic Innovation

Suggested running head: Experimental Use Licensing

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Abstract

We use a duopoly model of non-drastic innovation with production differentiation to examine the sequential pricing and simultaneous purchase decisions of an experimental use license in a three-stage game. In equilibrium, the technologically advantaged firm will purchase a license while the technologically disadvantaged does not (regardless of the order of pricing decisions). While efficiency requires both to purchase a license, this can only occur if the firm’s products have identical initial quality. Efficient sharing of intellectual property in R&D will occur if a research exemption is granted.

Keywords: Experimental Use, Innovation, Product Differentiation.

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1 Introduction

Innovations in the agricultural biotechnology industry can be characterized as having a cumulative element but in the presence of differentiated products. Firms seek to innovate creating new plant varieties with specific traits using existing plants and genetic code as the building blocks. Each firm controls certain seed varieties with traits aimed at specific soil and climate targets, such as roundup readiness or drought tolerance. Much R&D effort is expended trying to improve the yield (quality) of these varieties. Creation of new varieties with higher and higher yield (quality) require the appropriate building blocks. This type of quality improving innovation requires genetic diversity and much of the genetic building blocks in this industry are protected by a swath on patents and intellectual property protection limiting access.

A key feature of this type of cumulative innovation is that each firm’s variety progresses along its own quality ladder.\(^1\) Traditionally, the quality ladder model is used to characterize patent races whereby firms are racing to improve the existing product (move it to the next rung on the quality ladder) and garner monopoly profits as long as they possess the patent for the product currently at the highest rung on the quality ladder. We have in mind a very different industry structure as is modeled by Jackson and Smith (\(?\)add citation later?). They build on the models of Malla and Gray (2005) and Tangerás (2009) to construct a model of non-drastic innovation with two firms each moving along their own quality ladder as they produce outputs that compete in a Hotelling (1929) style model of product differentiation. As one firm improves its quality and the other does not, the innovating firm is able to increase profits and market share but does not monopolize the entire market. We find that this is a good foundation for modelling the types of quality improving innovations in agriculture biotechnology that are the focus of this paper.

Farms in different regions with different climates and soil have very different needs and demand seed with different traits. However, for any single farm deciding which seed variety

\(^{1}\)See Scotchmer (2004) for a summary of the quality ladder literature.
to purchase they ultimately don’t care about the trait target of the variety itself but what
the yield (quality) of the variety is for their soil/climate type. As one firm advances to a
higher yield (quality) step on their quality ladder some farms that previously purchased the
variety available from the other firm will switch to the innovated variety as it now provides
a greater profit opportunity to the farmer. It is very unlikely that the yield improvement is
great enough for all farm types for every farm to demand the improved variety. Many still
purchase the unimproved variety. This lack of innovation leading to market monopolization
necessitates the modeling of innovation as non-drastic.

Additionally, this newly discovered genetic code that improved the yield of a strain
targeted at one trait may also improve the yield of another strain targeted at a different
trait. Thus, this genetic discovery of one firm could improve the quality of strains owned
by other firms which can only be taken advantage of if access to the genetic material is
granted: either through a “research exemption” regime or the purchase of an experimental
use license.

Moschini and Yerokhin (2008b) provide a lengthy review of the intellectual property
right issues that permeate agricultural biotechnology. A detailed history of intellectual
property for agricultural biotechnology is also available in Wright and Pardey (2006) and
Smith (2008).2 In the U.S., biotechnology is patentable and there is no applicable research
exemption. If one biotechnology firm wants access to the patented genetic code owned
by a competitor for the purposes of research and development in the production of a new
patentable strain, it must purchase a license for experimental use. It is this feature of
experimental use licensing that is the primary focus of this research.

Our focus is similar to that of Moschini and Yerokhin (2008b) who focus on the effects
on innovation of different intellectual property rights regimes: with and without a “re-
search exemption” for experimental use in R&D. Under a regime of “research exemption”

2Additional papers that examine innovation incentives and intellectual property rights for agricultural
biotechnology are Moschini and Yerokhin (2008a), Koo and Wright (2010), Galushko, Gray, and Oikonomou
a firm can use proprietary intellectual knowledge in its’ R&D activities to create a newly patentable product idea without infringement concerns. However, if no such exemption exists, then such use constitutes infringement and risks legal action. In this case, the intellectual property of other firms can only be used for R&D if a license for experimental use is purchased. This is an important issue in many industries in which the product and intellectual property produced and owned by one firm could be an important input into the R&D activities of a competing firm. This is descriptive of the agricultural biotechnology industry that Moschini and Yerokhin (2008b) highlight. However, their model assumes that all market innovations move along one quality ladder with the innovating firm capturing the entire market share and the monopoly profits that go with it. This is inconsistent with the agricultural biotechnology industry in that newly innovated seed strains do not monopolize the market. The improved strains compete against existing strains for market share. Their structure neglects the reality that many innovations are quality improvements in existing products and not the creation of an entirely new product. Their model makes sacrifices in the strategic environment in order to maintain the tractability requirements of Markov Perfect Equilibrium which causes their analysis to underplay important aspects of the strategic environment that stem from licensing for experimental use.\(^3\) Our approach allows the strategy involved in experimental use licensing to take the center stage.

Product differentiation with quality improvements moving along a quality ladder has been looked at by Grossman and Helpman (1991). In their model each firm’s differentiated output follows its own stochastic progression along a quality ladder bringing about quality improvements in existing products. However, their main concerns are the implications that such innovations have for the long run rate of growth in the economy. As such, they ignore the intellectual property rights and licensing issues that are the focus of the present research.

Our model suppresses issues of patentability and breadth\(^4\) of an innovation to focus

\(^3\)Comments about licensing in Moschini and Yerokhin (2008b) are relegated to a small section without formal analysis.

\(^4\)Issues of patentability and breadth have long been the focus of the literatures. For examples see: Chang
on the competing interests of firms that can increase their own probability of innovation when an experimental use license is purchased from its rival. Likewise, firms can sell an experimental use license of its own product to its competitor. Doing so results in a gain in revenue from the sale yet increases the probability of the rival advancing along its own quality ladder and competing away future profits.\footnote{Schotchmer (1991) considers the effects of licensing in sequential innovation in the standard quality ladder. Early innovators may license to latter innovators who use their product in research and development. The ability for such agreements to be made depends on the breadth of patent protection afforded by law.} We model innovation as two firms move along their respective quality ladder as in Jackson and Smith (?add citation later?). In the model, the pricing and purchase of experimental use licenses follows a three-stage game. In the first two stages each firm successively sets the price of an experimental use license and in the third stage both firms simultaneously decide to purchase or not to purchase a license. We derive both the pricing decision and purchase decision for experimental use licensing and show that the equilibrium will not result in the efficient exchange of experimental use licenses unless the competing firms have identical levels of initial quality. This is the case regardless of whether the technologically advantaged prices before or after the technologically disadvantaged. If the intellectual property rights regime were modified to include an experimental use exemption, the efficient exchange intellectual property for the purposes of R&D would occur.

Section 2 analyzes the equilibrium of the experimental use licensing game. Section 3 discusses possible extensions to the framework for future research. Section 4 contains the conclusion.

## 2 Innovation with experimental use licensing

The licensing game is dynamic consisting of two time periods, $t = 0, 1$. In the first time period the game begins with firms, indexed by $i = \{a, d\}$, producing and selling their current product with quality $y^a_0$ and $y^d_0$. Each firm earns a profit which depends on the

\footnote{Hopenhayn and Mitchell (2001).}

current product quality of both firms. Each firm may innovate which improves the quality of its output in the following time period.

Firms interact strategically both in the product market and in the purchase and sale of experimental use licensing. Possession of an experimental use license for a rival firm’s product changes the distribution from which an innovation is stochastically drawn increasing the probability of advancement along the quality ladder. Firm \( a \) gets profits from their sales in the product market based on the current product qualities of both firms and any revenue or expenses from experimental use license sales and purchases. Finally, based on license purchase decisions, nature will select whether each firm innovates in the next time period which in turn determines profits earned in the product market in the terminal time period.

Let \( k_0 \) be the differential quality at the start of the game. We have purposefully labeled our two firms \( a \) and \( d \) to denote that firm \( a \) is the technological advantaged and firm \( d \) is the technological disadvantaged. We adopt this convention so that \( k_0 = y_a^0 - y_d^0 \geq 0 \) with the inequality being strict if the two firms begin the game at differing steps along their respective quality ladders. Each firm earns a payoff from its sales in the product market as it depends on the differential quality at that point in time. Product market profits depend only on differential quality, \( k_0 \) and \( k \), so we can write \( \pi^i(k_0) \) and \( \pi^i(k) \) for time period zero and one profits, respectively. Jackson and Smith (add citation later?) have derived profit equations in a parametric model of non-drastic innovation such as we describe here. We refer you to that work for in depth description and derivation of the profit equations as they depend on differential quality. The analysis we provide here is more general; we maintain only that the product market profit functions, \( \pi^a(k) \) and \( \pi^d(k) \), are convex and symmetric in that we can write \( -k = y_d^1 - y_a^1 \) and \( \pi^d(k) = \pi^a(-k) \).

We model innovation as a random process that depends on access to the rivals intellectual property. For notational simplicity we model innovation as it influences the differential quality of the two firms. At time zero \( \tilde{k} \) is a random variable with cumulative distribution
F and density f and support \( K \equiv \{ k_0 - \Gamma, k_0, k_0 + \Gamma \} \). \( \Gamma > 0 \) is the distance between rungs on each firm’s quality ladder. The realization, \( k \), of the random variable \( \tilde{k} \) determines time period one innovation by yielding the difference between time period 1 firm qualities with \( k = y_1^a - y_1^d \). If the quality differential narrows, as would happen when the advantaged firm does not innovate but the disadvantaged firm does, then \( k = k_0 - \Gamma \). If the quality differential widens, as would happen when the advantaged firm innovates but the disadvantaged firm does not, then \( k = k_0 + \Gamma \). Finally, if the quality differential stays the same, as would happen when either no firm innovates or when both firms simultaneously innovate, then the differential quality is unchanged with \( k = k_0 \).

In time period zero a three stage game is played. In the first stage of time period zero, one firm decides on a price to charge the other firm for an experimental use license to their product. Then, in the second stage, the other firm sets its price after observing the price set in stage one. Then, in the third stage each firm simultaneously decides whether or not to purchase the experimental use license from the other firm. The license has no effect on time period zero payoffs other than as a source of revenue or expenditure. Profits in time period one are determined solely by \( k \). However, nature will decide (probabilistically) whether each firm will innovate based upon the current value of \( k_0 \) and access to intellectual property as results from the combined license purchase decisions.

If \( i \) purchases a license from \( j \) then the indicator function \( B^i \) returns a value of 1; if \( i \) does not purchase a license from \( j \) then the function \( B^i \) takes on a value of zero. Nature chooses \( k \) according to a known cumulative distribution function \( F(k|k_0, B^a, B^d) = \int_{-\infty}^{x} f(x|k_0, B^a, B^d)dx \). Time period one is the terminal node with no license pricing or purchase decisions made. Payoffs are simply awarded based on the realized value of \( k \).

Holding the license purchase decision of one firm constant, the probability that the other firm will innovate is larger when it purchases a license. If both players innovate (or neither player innovates) the technological gap (and product market profits) is unchanged. This has consequences for the distribution of the technological gap as outlined in assumption
Assumption 2.1. \( F(k|k_0, 1, 0) > F(k|k_0, 0, 0) = F(k|k_0, 1, 1) > F(k|k_0, 0, 1) \)

The set-up of the model yields the following expected profits from the product market in time period one given the initial technology gap \( k_0 \) and license purchase decisions as

\[
E[\pi^i(k)|k_0, B^a, B^d] = \int_{k \in K} \pi^i(k)f(k|k_0, B^a, B^d)dk
\] (1)

As a direct result of assumption 2.1 we can write \( E[\pi^i(k)|k_0, 1, 1] = E[\pi^i(k)|k_0, 0, 0] = \pi^i \), for \( i = \{a, d\} \). Convexity of the profit functions give us the following inequalities\(^6\)

\[
E[\pi^a(k)|k_0, 1, 0] > \pi^a > E[\pi^a(k)|k_0, 0, 1]
\]

and

\[
E[\pi^d(k)|k_0, 0, 1] > \pi^d > E[\pi^d(k)|k_0, 1, 0].
\]

With \( \delta \) as the common discount factor and \( \rho^i \) being the price \( i \) sets for its license, the discounted payoffs to \( i \) if neither purchases a license is

\[
\pi^i(k_0) + \delta \pi^i
\]

and is

\[
\pi^i(k_0) + \delta \pi^i + \rho^i - \rho^i
\]

if both purchase a license.

We now proceed to solve for the best response functions for the license purchase decisions as they depend on the technology gap and license prices: \( k_0, \rho^a, \) and \( \rho^d \).

\(^6\)These inequalities are a direct application of Jensen’s inequality.
2.1 The Purchase Decision

As we proceed we make use of some simplifying notation. Define the discounted expected change in profit for $i$ as

$$\Delta \pi^i(x, y, u, v) = \delta E \left( \pi^i(k) | k_0, x, u \right) - \delta E \left( \pi^i(k) | k_0, y, v \right)$$

where $x$ and $y$ represent the advantaged’s, $a$, purchase decisions and $u$ and $v$ represent the disadvantaged’s, $d$, purchase decisions. Now we define $\kappa = 1, 0$ and $P = 1, 1$ and, finally, $N = 0, 0$. We can succinctly analyze the four separate cases as

$$\begin{align*}
\Delta \pi^i(\kappa, P) & \quad \Delta \pi^i(\kappa, N) \\
\Delta \pi^i(P, \kappa) & \quad \Delta \pi^i(N, \kappa)
\end{align*}$$

where $\kappa$ represents the change from not purchasing to purchasing and $P$ represents always purchasing and $N$ represents never purchasing.

Let $q^i \in [0, 1]$ be the probability that player $i$ purchases a license so that the payoff function can be written in the following general forms for each of the respective firms:

$$\Pi^a \left( \rho^a, \rho^d, q^a, q^d \right) =$$

$$\begin{align*}
\pi^a(k_0) + q^d \Delta \pi^a(N, \kappa) + \delta \pi^a + q^a \rho^a - q^a \rho^d \\
+ q^a q^d \Delta \pi^a(\kappa, P) + q^a (1 - q^d) \Delta \pi^a(\kappa, N)
\end{align*}$$

and
\[ \Pi^d(\rho^a, \rho^d, q^a, q^d) = \]
\[ \pi^d(k_0) + q^a \Delta \pi^d(\kappa, N) + \delta \pi^d + q^a \rho^d - q^d \rho^a \]
\[ + q^d q^a \Delta \pi^d(P, \kappa) + q^d (1 - q^a) \Delta \pi^d(N, \kappa) \]  

(3)

From these payoff functions the best response correspondence for the license purchase decision given the license prices can be simply derived. We make use of some additional simplifying notation. Define \( E^B_a(q^d) \) as the expected return to \( a \) from purchasing a license given the purchase decision of \( d, q^d \).

\[ E^B_a(q^d) = q^d \Delta \pi^a(\kappa, P) + (1 - q^d) \Delta \pi^a(\kappa, N) \]  

(4)

The expected return to \( d \) from purchasing a license given the purchase decision of \( a \) is analogously written as

\[ E^B_d(q^a) = q^a \Delta \pi^d(\kappa, P) + (1 - q^a) \Delta \pi^d(N, \kappa). \]  

(5)

The best response correspondence for firm \( i \) is given by:

\[ q^i(q^j) = \begin{cases} 
q^i = 1 & \text{if } E^B_i(q^i) \geq \rho^i \\
q^i = 0 & \text{if } E^B_i(q^i) \leq \rho^i \\
q^i \in [0, 1] & \text{if } E^B_i(q^i) = \rho^i
\end{cases} \]  

(6)

The best response correspondences for the license purchase decisions are compact valued and upper hemi-continuous guaranteeing the existence of a Nash equilibrium to the third
stage license purchase game for any given prices $\rho^a \geq 0$ and $\rho^d \geq 0$. If the prices charged by both firms are small compared to the change in expected profits brought about by the license purchase, then there will be an equilibrium in which both firm purchase an experimental use license for the others product. However, if either firm charges too high a price, the other firm won’t purchase the license. There are many possible equilibrium outcomes dependent on the prices of both licenses relative to the expected profits from purchasing. Each of the equilibrium is detailed in Proposition (1).

**Proposition 1.** Nash equilibrium to the license purchase game are characterized by the following conditions:

1. There is a pure strategy NE with $\{q^a, q^d\} = \{1, 1\}$ $\iff$ both
   
   (a) $\Delta \pi^a(\kappa, P) \geq \rho^d$  
   (b) $\Delta \pi^d(P, \kappa) \geq \rho^a$

2. There is a pure strategy NE with $\{q^a, q^d\} = \{0, 0\}$ $\iff$ both
   
   (a) $\Delta \pi^a(\kappa, N) \leq \rho^d$  
   (b) $\Delta \pi^d(N, \kappa) \leq \rho^a$

3. There is a pure strategy NE with $\{q^a, q^d\} = \{1, 0\}$ $\iff$ both
   
   (a) $\Delta \pi^a(\kappa, N) \geq \rho^d$  
   (b) $\Delta \pi^d(P, \kappa) \leq \rho^a$

4. There is a pure strategy NE with $\{q^a, q^d\} = \{0, 1\}$ $\iff$ both
   
   (a) $\Delta \pi^a(\kappa, P) \leq \rho^d$  
   (b) $\Delta \pi^d(N, \kappa) \geq \rho^a$

5. There is a mixed strategy NE with

   $$q^a = \frac{\rho^a - \Delta \pi^d(N, \kappa)}{\Delta \pi^d(P, \kappa) - \Delta \pi^d(N, \kappa)}$$
\[ q^d = \frac{\rho^d - \Delta \pi^a(\kappa, N)}{\Delta \pi^a(\kappa, P) - \Delta \pi^a(\kappa, N)} \]

if both

(a) \( \Delta \pi^d(P, \kappa) < \rho^a < \Delta \pi^d(N, \kappa) \)

(b) \( \Delta \pi^a(\kappa, P) < \rho^d < \Delta \pi^a(\kappa, N) \)

Proof. Follows directly from the best response function specified in equation (6). □

### 2.2 License Pricing: Advantaged Prices First

We now consider the research licensing game as a whole taking into account the complication of license pricing. We model pricing as a two stage process whereby one firm prices first with the other pricing subsequently after observing the price set by the other firm. In this section we consider that the technological advantaged, \( a \), prices first with the technological disadvantaged pricing second.

A strategy for the advantaged \( a \), \( s^a \), is a pair \( s^a = (q^a(\rho^a, \rho^d(\rho^a)), \rho^a) \) where \( q^a(\rho^a, \rho^d(\rho^a)) \) is the probability that \( a \) purchases \( f \)'s license and \( \rho^a \) is the price \( a \) charges \( f \) for \( a \)'s license. A strategy for the disadvantaged \( d \), \( s^d \), is a pair \( s^d = (q^d(\rho^d, \rho^a(\rho^d)), \rho^d(\rho^d)) \) where \( q^d(\rho^d, \rho^a(\rho^d)) \) is the probability that \( d \) purchases \( a \)'s license and \( \rho^d \) is the price \( d \) charges \( a \) for \( d \)'s license. Because we focus our solution on subgame perfect equilibrium we require that each player believes that the other will always make purchase decisions according to her best response function, \( q^f(\rho^f, \rho^d) \).

**Definition 2.1.** A subgame perfect equilibrium to the pricing game is a strategy profile, \( s \), such that the following conditions are met:

1. \( q^a \) and \( q^d \) are a Nash Equilibrium to the license purchase game given \( \rho^a \) and \( \rho^d(\rho^a) \).

2. \( \rho^d(\rho^a) \) must be optimal given purchase strategies \( q^a \) and \( q^d \) and the price \( \rho^a \) set by the advantaged.
3. \( \rho^a \) must be optimal given purchase strategies \( q^a \) and \( q^d \) and the pricing strategy of \( f \), \( \rho^d(\rho^a) \).

We proceed using backward induction to find subgame perfect equilibria by first finding the optimal pricing strategy for the disadvantaged given the price set by the advantaged. Proposition 2 shows \( \rho^d(\rho^a) \) will take one of two values depending on whether \( \rho^a \) is above or below a threshold. If \( \rho^a \) is sufficiently large the disadvantaged sets a price for its license whereby only the technological advantaged will purchase a license. However, with \( \rho^a \) sufficiently low the disadvantaged will set its price at a level so that both purchase a license.

**Proposition 2.** The best response function for the disadvantaged is

\[
\rho^d(\rho^a) = \begin{cases} 
\Delta \pi^a(\kappa, N) & \text{if } \rho^a > \Delta \pi^d(P, \kappa) \\
\Delta \pi^a(\kappa, P) & \text{if } \rho^a \leq \Delta \pi^d(P, \kappa)
\end{cases}
\]

*Proof. See Appendix.*

Given this best response function for the disadvantaged, the technological advantaged can choose to either price low so that both end up purchasing a license or price high so that only the advantaged purchases having effectively priced the disadvantaged out of the license market. Proposition 3 shows that it is in the best interest of the advantaged to price at a high level.

**Proposition 3.** The optimal price for the advantaged is any \( \rho^a(k_0) \) such that \( \rho^a(k_0) > \Delta \pi^d(P, \kappa) \).

*Proof. See Appendix.*

Now that we have derived the purchase decisions and the optimal pricing decisions of the technological advantaged and disadvantaged, we can focus on equilibrium. While this
game does have multiple prices the advantaged could set in equilibrium, all such equilibria produce the same equilibrium outcome and payoffs. The advantaged prices too high for the disadvantaged to purchase its license while the disadvantaged sets its price at the highest possible level which the advantaged is willing to purchase at. This is summarized in Theorem 1.

**Theorem 1.** If the technological advantaged price’s first, then all subgame perfect equilibria to this game have the same outcome that involves the advantaged purchasing a license at a price of \( \rho^d = \Delta \pi^a(\kappa, N) \) and the advantaged setting a price \( \rho^a > \Delta \pi^d(P, \kappa) \) so that the disadvantaged does not purchase a license.

*Proof.* Follows directly from Propositions 1, 2, and 3. \( \square \)

### 2.3 License Pricing: Disadvantaged Prices First

Next we consider the case where the technological disadvantaged prices in the first stage with the advantaged pricing in the second second stage after observing the price set by the disadvantaged. In the third stage both players simultaneously make purchase decisions. This switch of order requires a slight change in strategy and equilibrium definitions.

A strategy for the advantaged, \( s^a \), is a pair \( s^a = (q^a(\rho^a(\rho^d)), \rho^a(\rho^d)) \) where \( q^a(\rho^a(\rho^d)) \) is the probability that \( a \) purchases \( f \)’s license and \( \rho^a \) is the price \( a \) charges \( f \) for \( a \)’s license. A strategy for the disadvantaged, \( s^d \), is a pair \( s^d = (q^d(\rho^d(\rho^a)), \rho^d(\rho^a)) \) where \( q^d(\rho^d(\rho^a)) \) is the probability that \( d \) purchases \( l \)’s license and \( \rho^d \) is the price \( d \) charges \( l \) for \( d \)’s license. Since we focus our solution on subgame perfect equilibrium we require that each player believes that the other will always make purchase decisions according to her best response function \( q^i(\rho^i, \rho^d) \).

**Definition 2.2.** A subgame perfect equilibrium to the re-ordered two time period pricing game is a strategy profile \( s \) such that the following conditions are met:
1. \( q^a \) and \( q^d \) are a Nash Equilibrium to the license purchase game given \( \rho^a(\rho^d) \) and \( \rho^d \).

2. \( \rho^d \) must be optimal given purchase strategies \( q^a \) and \( q^d \) and the and the pricing strategy of the advantaged, \( \rho^a(\rho^d) \)

3. \( \rho^a(\rho^d) \) must be optimal given purchase strategies \( q^a \) and \( q^d \) and the price \( \rho^d \) set by the disadvantaged.

Again, this game is solved using backward induction to find subgame perfect equilibrium. First we find the optimal pricing strategy for the advantaged given the price set by the disadvantaged. If the disadvantaged has priced its license sufficiently low then the advantaged will set a price price that is high enough so that only the advantaged will make a license purchase. However, if the disadvantaged has set a high price, the advantaged will set its price in such a manner that neither will purchase a license. This result is given in Proposition 4.

**Proposition 4.** The best response function for the advantaged is

\[
\rho^a(\rho^d) = \begin{cases} 
\Delta \pi^d(N, \kappa) & \text{if } \rho^d \geq \Delta \pi^a(\kappa, N) \\
\Delta \pi^d(P, \kappa) & \text{if } \Delta \pi^a(\kappa, N) > \rho^d
\end{cases}
\]

*Proof.* See Appendix. \(\square\)

Given the best response function of the advantaged’s license pricing as a function of the disadvantaged’s license price, the disadvantaged essentially has a decision to either set its price in a manner that will ultimately result in only the advantaged purchasing a license or setting its price such that neither the advantaged nor the disadvantaged purchase a license. The disadvantaged gets the higher payoff by pricing so that the advantaged will purchase its license. The price the disadvantaged sets is given in Proposition 5.

**Proposition 5.** The optimal price for the disadvantaged is \( \rho^d = \Delta \pi^a(\kappa, P) \).
Proof. See Appendix.

We now have all of the components needed to make a statement about the subgame perfect equilibria to the license pricing game with the disadvantaged pricing first. Again, there are many equilibria to this game as there are many prices that the advantaged could set that would yield identical payoffs. However, each of these equilibria have the same outcome with only the advantaged purchasing a license as is summarized in Theorem 2.

**Theorem 2.** If the technological disadvantaged prices first, then all subgame perfect equilibria to this game have the same outcome that involves the disadvantaged setting a a price of \( \rho^d = \Delta \pi^a(\kappa, P) \) and the advantaged setting a price \( \rho^a > \Delta \pi^d(P, \kappa) \) so that the disadvantaged does not purchase a license but the advantaged will.

*Proof.* Follows from propositions 1, 4, and 5.

### 2.4 Discussion

Theorems 1 and 2 are strikingly similar. Whether the advantaged or disadvantaged prices first has no bearing on the final license purchase decisions other than the price paid by the advantaged for a license to use the technology of the disadvantaged. In both scenarios only the advantaged purchases a license and the advantaged actually pays a lower price for the license it purchases when the disadvantaged prices first.

### 2.5 A Technological Tie

Now suppose that both firms start the game at an identical rung on their respective quality ladder, i.e. \( k_0 = 0 \). In this case neither can be characterized as a advantaged or a disadvantaged so it is unnecessary to list identity in the order of play. Without loss of generality we let player \( i \) price first followed by player \( j \). The best response pricing correspondence of
player $j$ as it depends on the price set by player $i$ is the same as that found in proposition 2, replacing $a$ with $i$ and $d$ with $j$.

The next step is to identify the optimal pricing strategy of player $i$ given the best response of $j$. The optimal pricing strategy for $i$ is given in proposition 6 below which is quite similar to the result in proposition 3 except that the inequality is now weak. The player pricing first can either set the price such that the equality is satisfied resulting in both players purchasing a license or it can be set the price high so that the player pricing first is the only one who will purchase a license.

**Proposition 6.** The optimal price for player $i$ is $\rho^i(k_0) \geq \Delta \pi^j(P, \kappa)$.

*Proof.* See Appendix.

Using these optimal pricing strategies it is now possible to characterize equilibrium for the case of a technological tie in theorem 3 below. The theorem shows that two possible outcomes could occur in a subgame perfect equilibrium. Either the equilibrium is practically identical to that in Theorem 2 with the player pricing first purchasing a license but the player pricing second not purchasing a license or we have the outcome of both players purchasing a license.

**Theorem 3.** When there is no technological advantage, $k_0 = 0$, and player $i$ prices before $j$, then there are two possible outcomes in a subgame perfect equilibrium.

1. Player $i$ purchases a license at a price of $\rho^i = \Delta \pi^i(\kappa, N)$ and $i$ setting a price $\rho^i > \Delta \pi^j(P, \kappa)$ so that $j$ does not purchase a license.

2. Player $i$ purchases a license at a price of $\rho^i = \Delta \pi^i(\kappa, N)$ and $i$ setting a price $\rho^i = \Delta \pi^j(P, \kappa)$ so that both $i$ and $j$ purchase a license.

*Proof.* Follows directly from Propositions 1, 2, and 6.
2.6 Efficient Licensing

In order to consider the welfare effects of innovation we need to consider the product market once again. Our model of the product market is that of Jackson and Smith (add citation later?) which, in addition to deriving the product market profit function we make use of, also gives welfare implications of innovation. Proposition 4 in that paper shows that welfare increases the most when both firms innovate rather than only one firm innovating.

Simultaneous innovation by both firms increases total welfare by the largest amount so that efficient experimental use licensing requires each firm to buy a license from the other. This maximizes the probability that both firms will innovate. Unfortunately, this pattern of licensing is rarely present in equilibrium. In fact, the only case that both firms could both purchase an experimental use license is when both firms have identical initial product quality. Even then, the equilibrium is not unique as there is also an equilibrium where the firm that prices its license first is the only firm that purchases an experimental use license.

What would be the appropriate policy to combat this lack of efficiency in innovation resulting from too little experimental use licensing? The main policy tool at play is the intellectual property rights regime itself. We considered the regime in which each firm has the right not only to sell its product exclusively but also possesses the exclusive right to use its product in research and development. An alternative property right regime is to maintain exclusivity in product sales but give a research exemption to allow the free use of existing products in the research and development activities of all firms. This regime allows research and development to move forward uninhibited by the sluggish sales of experimental use licenses and would thus be efficient. However, our analysis does ignore the decision to conduct R&D itself. With a research exemption the incentive to conduct R&D in the first place will certainly be diminished.
3 Extensions

3.1 Large Innovations

In the analysis we present above we have considered the possibility of patentable innovations that are small in the sense that they do not allow the innovator to capture the entire market share. In this section we discuss the effect on experimental use licensing of a large innovation which does allow the innovator to capture the entire market.

In this case, the innovation is of such a large magnitude that all consumers will choose to purchase the product of the innovating firm. To keep matters simple, we assume that in this case the non-innovating firm goes out of business resulting in profits of zero in the subsequent period while the innovating firm captures monopoly profits of $M > \frac{1}{4}\pi(k)$.

Whenever a license is sold in our model the selling party charges a price such that the entire discounted expected gain in profits from the innovation are appropriated to the seller. This leaves the purchaser indifferent between buying the license or not. The seller is only willing to make such a sale if this sales price is sufficient to cover the discounted expected loss it would suffer from falling behind technologically. Now that the sale of the license increases the probability of the advantaged innovating and thus the disadvantaged going out of business, the sale price of the license will have to be sufficiently large to cover all of the expected loss. Such a license will only be sold if the monopoly profits are sufficiently large. The advantaged will still find itself in a situation whereby it is unable to sell a license to the disadvantaged.

Innovation which leads to monopoly power decreases the parameter space for which any experimental use license can be sold and does not allow for both firms to buy a license from the other in any equilibrium.
3.2 Complementary Research and Development

A key limitation of the model presented here is that the decision of how much to invest in research and development itself is ignored. We hold research and development expenditure and effort constant isolating only the marginal impact of experimental use licensing on innovation. A richer model would make the decision of research and development expenditures endogenous. A key question is whether complementarity between internal research and development efforts and possession of an experimental use license increases the incidence under which experimental use licenses can be sold. Allowing innovation to occur with increasing probability with increased internal R&D expenditure reduces a firm's reliance on possession of an experimental use license for the purposes of innovation, the price that is required by a seller of an experimental use license is likely to be lower. However, with high levels of complementarity between internal R&D and experimental use licensing (large investment in internal R&D can increase the degree to which an experimental use license increases the probability of innovation) the value of the license to the purchaser increases as does the price required by a license seller. These effects cannot be intuited without additional modeling.

3.3 Extended Time Horizon

Our model was created to focus attention on the strategic interaction stemming from experimental use license pricing and purchase decisions when access to a competing firm's output for experimental research increases the probability of innovation. In keeping this focus we only consider a world with two time periods. One direction for future research is to consider the effect of a longer time horizon on the licensing decisions. A longer time horizon can have large consequences as it becomes possible for innovation to cause lead switching. If the disadvantaged is able to innovate in multiple time periods while the advantaged fails to innovate, eventually the disadvantaged’s quality level will overtake that of the advantaged. This possibility will increase the value of an experimental use license to the disadvantaged
but it will simultaneously raise the price that the advantaged would require to make such a sale.

4 Conclusion

The nature of technological innovation for products in many markets follow the model of sequential innovation as set forth by the quality ladder model. However, a limiting assumption of the quality ladder model is that when a firm is able to make a patentable innovation it is then able to monopolize the market. Yet, most innovated products compete alongside older generations of products from other firms. The innovation does give the innovating firm an advantage but it generally doesn’t eliminate all competition.

We develop a model of sequential innovation whereby two firms produce differentiated products for a market in which they engage in price competition for market share. Innovation by a firm results in a quality improvement of the existing product. Additionally, firms may buy and sell a license for the experimental use of each others product. Buying a license to use a competitors product in research and development increases the probability of an innovation occurring. This complicates the strategic environment as selling a license has two effects: a direct increase in revenue due to the sale and an increased probability of the competitor innovating which reduces profitability. We derive equilibrium pricing and purchase strategies for experimental use licenses.

In equilibrium play with one firm having a technological advantage, the order of play has little bearing on the equilibrium. The technological advantaged will purchase an experimental use license from the disadvantaged but the disadvantaged will fail to purchase a license from the advantaged. The largest increase in welfare occurs when both firms produce an innovation. Because the probability of both firms producing an innovation is the largest when both firms purchase an experimental use license, efficient experimental use licensing requires both firms to buy and sell a license. In a regime without an experimental
use exemption, the only condition under which efficient license sales can occur is when the firms have products with identical initial quality.
References


5 Appendix

5.1 Licensing Proofs

Proof. Proof of Proposition 2

1. Case 1: Let $\Delta \pi^d(N, \kappa) > \Delta \pi^d(P, \kappa) > \rho^a$.

Because players must play a NE in the third stage of the game, we can look to Proposition 1 to see where play in 3rd stage will end up dependant on second stage play. Nash equilibrium play in the purchase game will result in either equilibrium (1) or (4) from proposition Proposition 1.

It follows that the payoff in equilibrium (1) is larger than the payoff in equilibrium (4) if

$$\rho^d \geq \delta \left( E(\pi^d(k)|k_0, 0, 1) - \pi^d \right) > 0.$$

From 1(a) in Proposition 1 we know that there is an upper bound on the price $\rho^d$, $\Delta \pi^a(\kappa, P) \geq \rho^d$. We also know from the profit function that the disadvantaged prefers to sell the license at the largest price possible: the upper bound. Therefore, it remains to be shown that

$$\rho^d = \Delta \pi^a(\kappa, P) \geq \delta \left( E(\pi^d(k)|k_0, 0, 1) - \pi^d \right).$$

This inequality can be rewritten as

$$\pi^a - E(\pi^a(k)|k_0, 0, 1) \geq E(\pi^d(k)|k_0, 0, 1) - \pi^d.$$

It follows from assumption 2.1 and the convexity of the profit functions, $\pi^i(k)$, that whenever $k_0 < 3\tau$ it must be true that $\pi^a - E(\pi^a(k)|k_0, 0, 1) > E(\pi^d(k)|k_0, 0, 1) - \pi^d$. Therefore the payoff to $d$ must be greater in (1) than in (4).
We can then conclude that the disadvantaged optimally prices at \( \rho^d = \Delta \pi^a(\kappa, P) \) whenever the advantaged has priced such that \( \Delta \pi^d(P, \kappa) > \rho^a \).

Case 2: Let \( \rho^a > \Delta \pi^d(N, \kappa) > \Delta \pi^d(P, \kappa) \). Nash strategies in the purchase game require the pricing by the disadvantaged to force the purchase equilibrium into either (2) or (3) in Proposition 1.

The payoff to (3) is larger than (2) if

\[
\rho^d \geq -\Delta \pi^d(\kappa, N).
\]

In the Nash equilibrium (3) \( d \) wishes to make the price as large as possible setting \( \rho^d = \Delta \pi^a \). It remains to be shown that

\[
\rho^d = \Delta \pi^a(\kappa, N) \geq -\Delta \pi^d(\kappa, N)
\]

which reduces to

\[
\pi^a(k_0) + \pi^d(k_0) \leq E(\pi^a(k)|k_0, 1, 0) + E(\pi^d(k)|k_0, 1, 0).
\]

It follows from assumption 2.1 and the convexity of the profit functions, \( \pi^i(k) \), that whenever \( k_0 < 3\tau \) it must be true that \( \pi^a - E(\pi^a|1, 0) < E(\pi^d|1, 0) - \pi^d \) which proves the result.

Case 3: Let \( \Delta \pi^d(N, \kappa) \geq \rho^a > \Delta \pi^d(P, \kappa) \). Nash equilibrium play in the purchase game requires that the pricing decision of the disadvantaged forces the purchase equilibrium into either (3), (4) or (5) of Proposition 1. First, we compare (3) and (4).

The payoff to the Nash equilibrium in (3) is larger than in (4) if

\[
\rho^d \geq \Delta \pi^d(N, \kappa) - \Delta \pi^d(\kappa, N) - \rho^a.
\]
Because $\Delta \pi^d(N, \kappa) \geq \rho^a > \Delta \pi^d(P, \kappa)$ it follows that

$$\Delta \pi^d(N, \kappa) - \Delta \pi^d(\kappa, N) - \Delta \pi^d(P, \kappa) > \Delta \pi^d(N, \kappa) - \Delta \pi^d(\kappa, N) - \rho^a \geq -\Delta \pi^d(\kappa, N).$$

We also know that $d$ will make the price as large as possible, $\rho^d = \Delta \pi^a(\kappa, N)$.

If we can show that

$$\Delta \pi^a(\kappa, N) \geq \Delta \pi^d(N, \kappa) - \Delta \pi^d(\kappa, N) - \Delta \pi^d(P, \kappa)$$

then we will have proven that the payoff from equilibrium (3) is strictly greater than equilibrium (4).

The equation above reduces to

$$\Delta \pi^a(\kappa, N) \geq \delta E(\pi^d(k)|k_0, 0, 1) - \delta \pi^d = \Delta \pi^d(N, \kappa)$$

which must be true because $a$ is the technological advantaged (the inequality reduces to equality in the case of a technological tie) and the expected gain to the advantaged from buying a license when the disadvantaged doesn’t purchase is larger than the expected gain to the disadvantaged from buying a licence when the advantaged doesn’t due to assumption 2.1 and the convexity of the profit functions, $\pi^i(k)$.

Next we compare the payoff to the disadvantaged from forcing the equilibrium into (3) versus forcing the equilibrium into the mixed strategy equilibrium (5). If the disadvantaged forces the equilibrium into (3) she will do so setting the price as high as possible with $\rho^d = \Delta \pi^a(\kappa, N)$. If the payoff to (3) is to be larger the following condition must hold where $\rho^d$ is the price the disadvantaged charges in equilibrium
\[ \Delta \pi^a(\kappa, N) > (q^a - 1)\Delta \pi^d(\kappa, N) + q^a \rho^d - q^d \rho^a + q^a q^d \Delta \pi^d(P, \kappa) + q^d (1 - q^a) \Delta \pi^d(N, \kappa). \]

Because \( \Delta \pi^d(N, \kappa) > \rho^a > \Delta \pi^d(P, \kappa) \) and if the disadvantaged pushes equilibrium into (5) then \( \rho^d > \Delta \pi^d(I(K, N), \kappa) \), after some algebraic manipulation the above equation can be shown to be true if we can show that

\[ \Delta \pi^a(\kappa, N) \geq -\Delta \pi^d(\kappa, N) + q^d (\Delta \pi^d(N, \kappa) - \Delta \pi^d(P, \kappa)). \]

Because \( 0 < q^d < 1 \) in (5) it follows that

\[ \Delta \pi^d(N, \kappa) \geq \Delta \pi^d(N, \kappa) > \Delta \pi^d(P, \kappa). \]

We know that \( \Delta \pi^a(\kappa, N) \geq \Delta \pi^d(N, \kappa) \) which establishes the result.

Therefore, it is optimal for the disadvantaged to price the license at \( \rho^d = \Delta \pi^a(\kappa, N) \) when the advantaged has priced such that \( \Delta \pi^d(N, \kappa) \geq \rho^a > \Delta \pi^d(P, \kappa) \).

\[ \square \]

**Proof.** Proof of Proposition 3

If the advantaged prices such that \( \rho^a > \Delta \pi^d(P, \kappa) \) then given Propositions 2 and 1 the NE of the buy game will end up in (3) of Proposition 1. Using the disadvantaged’s best response function we see that \( \rho^d = \Delta \pi^a(\kappa, N) \). This results in a payoff to the advantaged of

\[ \pi^a(k_0) + \delta \pi^a(k_0). \]

If the advantaged prices such that \( \rho^d(k_0) \leq \Delta \pi^d(P, \kappa) \) then the Nash Equilibrium of
the buy game will be in (1) from Proposition 1 resulting in a payoff to the advantaged of

$$\pi^a(k_0) + \Delta \pi^a(N, \kappa) + \delta \pi^a(k_0) + \rho^a.$$ 

The payoff to the advantaged is bigger in (3) than in (1) if

$$-\Delta \pi^a(N, \kappa) > \rho^a.$$ 

Since the advantaged will always want to capture the largest price possible in (1) we have \(\rho^a = \Delta \pi^d(P, \kappa)\), which reduces our equation to \(-\Delta \pi^a(N, \kappa) > \Delta \pi^d(P, \kappa)\) and even further to

$$\pi^a - E(\pi^a(k)|k_0, 0, 1) > \pi^d - E(\pi^d(k)|k_0, 1, 0).$$

This must be true given \(k_0 < 3t\), assumption 2.1, and the convexity of the profit functions, \(\pi^i(k)\).

**Proof.** Proof of Proposition 4

1. Let \(\Delta \pi^a(\kappa, N) \geq \Delta \pi^a(\kappa, P) \geq \rho^d.\)

Since \(k_0 < 3t\) the convexity of the profit functions and assumption 2.1 imply that the advantaged would like the technological gap to get larger while the disadvantaged would like the gap to get smaller. However, the gain to the advantaged from making the gap larger is bigger than the gain to the disadvantaged from making the gap smaller. Likewise the loss to the advantaged to falling behind is bigger than the gain to the disadvantaged from closing the gap. Therefore, the disadvantaged must trade off between income from a license purchase and the reduction in profit from the gap getting larger.

With a low price from the disadvantaged, the advantaged will buy no matter what the
disadvantaged’s purchase decision is. This puts the advantaged in a choice between equilibrium (1) or (3) from Proposition 1.

Suppose that the payoff in (1) is at least as large as the payoff in (3) for the advantaged. Then

\[ \Delta^a(\kappa, N) - \Delta^a(\kappa, P) - \Delta^a(N, \kappa) \leq \rho^a \]

and \( \rho^a \leq \Delta^d(P, \kappa) \) or equivalently

\[ \delta (E(\pi^a(k)|k_0, 1, 0) - E(\pi^a(k)|k_0, 1, 1)) \leq \rho^a \leq \delta \left( E(\pi^d(k)|k_0, 1, 1) - E(\pi^d(k)|k_0, 1, 0) \right). \]

There does not exist a \( \rho^a \) that can make this true because the convexity of the profit functions and assumption 2.1 imply that

\[ \delta (E(\pi^a(k)|k_0, 1, 0) - E(\pi^a(k)|k_0, 1, 1)) > \delta \left( E(\pi^d(k)|k_0, 1, 1) - E(\pi^d(k)|k_0, 1, 0) \right). \]

Therefore the best response of the advantaged is to set the price high enough that the disadvantaged will not purchase a license. This requires a price such that \( \rho^a > \Delta^d(P, \kappa) \).

2. Suppose \( \Delta^a(\kappa, N) \geq \rho^d > \Delta^a(\kappa, P) \).

In this instance, the advantaged can force the game into a purchasing Nash Equilibrium of either (3), (4) or (5) from Proposition 1 by choosing her price accordingly. First we compare the expected payoffs to the advantaged from forcing the game into either equilibrium (3) or (4). The expected payoff to the advantaged from pricing into (3) is greater than pricing into (4) if

\[ \Delta^a(\kappa, N) - \rho^d > \Delta^a(N, \kappa) + \rho^a. \]
Since in (3) we have $\Delta \pi^a(\kappa, N) \geq \rho^d$ and in (4) we have $\Delta \pi^d(N, \kappa) \geq \rho^a$ we know that

$$\Delta \pi^a(\kappa, N) + \Delta \pi^d(N, \kappa) \geq \rho^a + \rho^d.$$ 

If we can show that $\Delta \pi^a(\kappa, N) - \Delta \pi^a(N, \kappa) > \Delta \pi^a(\kappa, N) + \Delta \pi^d(N, \kappa)$ then we have established that the payoff under (3) is higher than (4). This condition reduces to

$$-\Delta \pi^a(N, \kappa) > \Delta \pi^d(N, \kappa).$$ 

This reduces further to

$$\pi^a - E(\pi^a(k)|k_0, 0, 1) > E(\pi^d(k)|k_0, 0, 1) - \pi^d.$$ 

It follows from assumption 2.1 and the convexity of the profit functions, $\pi^i(k)$, that whenever $k_0 < 3\tau$ this inequality must hold.

Now we compare the expected payoff to the advantaged from forcing the purchasing game into equilibrium (3) or equilibrium (5) from Proposition 1.

The payoff to the technological advantaged is bigger in (3) than in (5) if the following holds.

$$\Delta \pi^a(\kappa, N) \geq q^d \Delta \pi^a(N, \kappa) + q^d \rho^a + (1-q^a)\rho^d + q^a q^d \Delta \pi^a(\kappa, P) + q^a (1-q^d) \Delta \pi^a(\kappa, N) \equiv (\star)$$ 

Because in (5) both $\rho^d < \Delta \pi^a(\kappa, N)$ and $\rho^a < \Delta \pi^d(N, \kappa)$, we have

$$q^d \Delta \pi^a(N, \kappa) + q^d \Delta \pi^d(N, \kappa) + (1-q^a)\Delta \pi^d(\kappa, N) + q^a q^d \Delta \pi^a(\kappa, P) + q^a (1-q^d) \Delta \pi^a(\kappa, N)) > (\star).$$ 

If we can show that

$$-\Delta \pi^d(N, \kappa) \geq \Delta \pi^a(N, \kappa) + q^a (\Delta \pi^a(\kappa, P) - \Delta \pi^a(\kappa, N))$$ 

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then we will have established the result.

Because $0 < q^a < 1$, $-\Delta \pi^d (N, \kappa) \geq \Delta \pi^a (N, \kappa)$, and $\Delta \pi^a (\kappa, P) \leq \Delta \pi^a (\kappa, N)$, the inequality above holds and it is true that the payoff in equilibrium $(3)$ is greater than the payoff in $(5)$ for the advantaged.

Therefore, the best response in this case is to set the price at any price high enough to force the equilibrium into $(3)$, $\rho^a > \Delta \pi^d (P, \kappa)$.

3. Suppose $\rho^d \geq \Delta \pi^a (\kappa, N) \geq \Delta \pi^a (\kappa, P)$.

The advantaged’s price strategy will cause Nash Equilibrium purchase strategies to result in either equilibrium $(2)$ or $(4)$ from proposition 1. The expected payoff to the advantaged from pricing into $(4)$ is greater than pricing into $(2)$ if

$$\rho^a \geq -\Delta \pi^a (N, \kappa).$$

If the pricing forces the buy game into $(4)$ then the advantaged will set the price such that $\rho^a = \Delta \pi^d (N, \kappa)$ as this is the upper bound on what the disadvantaged is willing to pay. Thus, the above condition can be rewritten as

$$\delta \left( E(\pi^d (k)|k_0, 0, 1) - E(\pi^d (k)|k_0, 0, 0) \right) \geq \delta \left( E(\pi^a (k)|k_0, 0, 0) - E(\pi^a (k)|k_0, 0, 1) \right)$$

This can never be true as a result the lead being strict, $k_0 > 0$, along with assumption 2.1 and convexity of the profit functions. Therefore, it must be the case that the payoffs under $(2)$ are greater than $(4)$ and the advantaged will price so that neither party will make a purchase.

We therefore have that the best response for the advantaged is to set the price high enough that the disadvantaged will not make a purchase, that is any $\rho^a > \Delta \pi^d (N, \kappa)$. 

\[\square\]
Proof. Proof of Proposition 5

If the disadvantaged prices so that \( \rho^d \geq \Delta \pi^a(\kappa, N) \) then the advantaged will price such that \( \rho^a \geq \Delta \pi^d(N, \kappa) \). In this scenario the buy equilibrium is in (2) from Proposition 1 in which neither buys a license. The payoff to the disadvantaged is \( \pi^d(k_0) + \delta E[\pi^d(k)|k_0, 0, 0] \).

If the disadvantaged prices so that \( \rho^d < \Delta \pi^a(\kappa, P) \) then the advantaged will price such that \( \rho^a > \Delta \pi^d(P, \kappa) \). In this scenario the buy equilibrium is in (3) from Proposition 1 in which the advantaged buys but the disadvantaged does not. The disadvantaged will want the highest price possible and will hence set \( \rho^d = \Delta \pi^a(\kappa, P) \). The payoff to the disadvantaged is \( \pi^d(k_0) + \delta E[\pi^d(k)|k_0, 0, 0] + \Delta \pi^d(\kappa, N) + \Delta \pi^a(\kappa, P) \).

Due to the convexity of the profit functions and assumption 2.1, we know that \( \Delta \pi^d(\kappa, N) + \Delta \pi^a(\kappa, P) > 0 \) so that the payoff to the disadvantaged is largest when the disadvantaged prices such that equilibrium will end up in (3) with \( \rho^d = \Delta \pi^a(\kappa, P) \).

Proof. Proof of Proposition 6 If \( i \) prices such that \( \rho^i(k_0) > \Delta \pi^i(P, \kappa) \) then the Nash Equilibrium of the buy game will be in (3) from Proposition 1. From \( j \)'s best response function we see that \( \rho^j = \Delta \pi^i(\kappa, N) \). This results in a payoff to \( i \) of

\[
\pi^i(k_0) + \delta \pi^i(k_0).
\]

If \( i \) prices such that \( \rho^i(k_0) \leq \Delta \pi^i(P, \kappa) \) then the Nash Equilibrium of the buy game will be in (1) from Proposition 1. This results in a payoff to \( i \) of

\[
\pi^i(k_0) + \Delta \pi^i(N, \kappa) + \delta \pi^i(k_0) + \rho^i.
\]
The payoff to $i$ is at least as big in (3) than in (1) if

$$-\Delta \pi^i(N, \kappa) \geq \rho^i.$$ 

Since $i$ will always want to capture the largest price possible in (1) we have $\rho^i = \Delta \pi^j(P, \kappa)$, which reduces our equation to

$$-\Delta \pi(N, \kappa) \geq \Delta \pi^j(P, \kappa).$$

After substituting we have

$$\pi^i(k_0) - E(\pi^i(k)|k_0, 0, 1) \geq \pi^j(k_0) - E(\pi^j(k)|k_0, 1, 0).$$

Which holds with equality for the case of $k_0 = 0$. 

$\square$